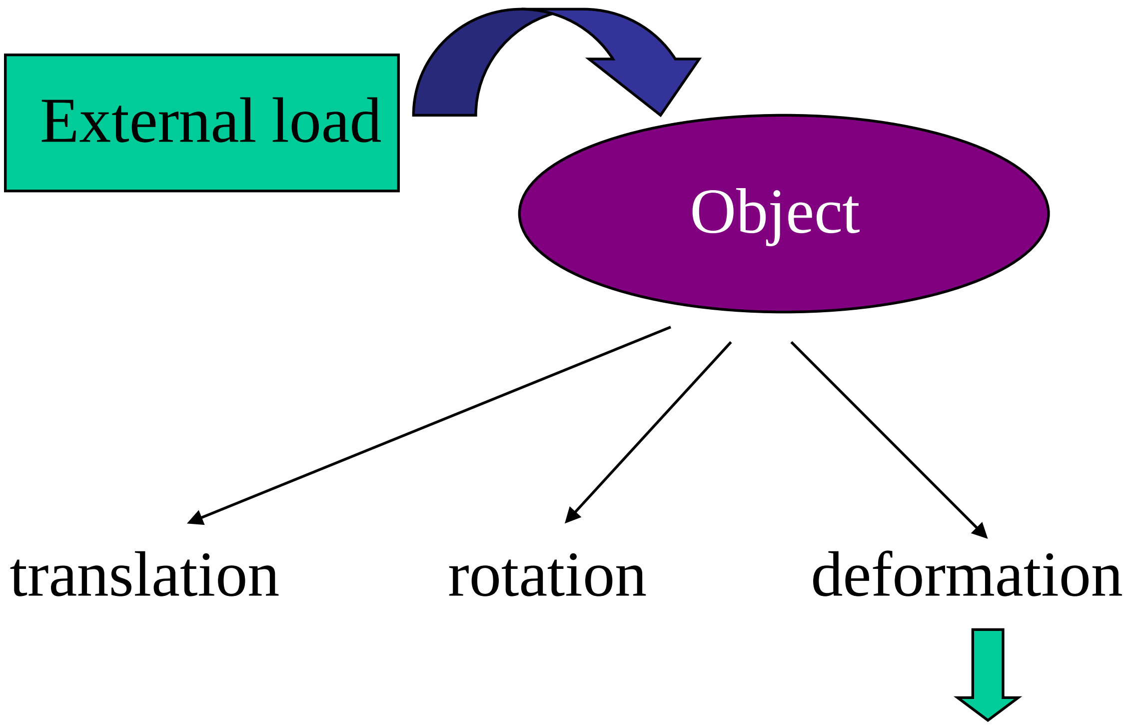
Module-4

Mechanical Properties of Metals

Contents

Mechanical loads - Deformation



distortion – change in shape

dilatation – change in size

Deformation – function of time?

Temporary / recoverable

time independent –

elastic

time dependent –

anelastic (under load),

elastic aftereffect (after removal of load)

Permanent

time independent –

plastic

time dependent –

creep (under load),

combination of recoverable and permanent, but time dependent – visco-elastic

Engineering Stress – Engineering Strain

- Load applied acts over an area.
- Parameter that characterizes the load effect is given as load divided by original area over which the load acts. It is called *conventional stress* or *engineering stress* or simply *stress*. It is denoted by s.
- Corresponding change in length of the object is characterized using parameter – given as per cent change in the length – known as *strain*. It is denoted by e.

$$s = \frac{P}{A_0}, e = \frac{L - L_0}{L_0}$$

- As object changes its dimensions under applied load, engineering stress and strain are not be the true representatives.

True Stress – True Strain

- *True* or *Natural* stress and strain are defined to give true picture of the instantaneous conditions.

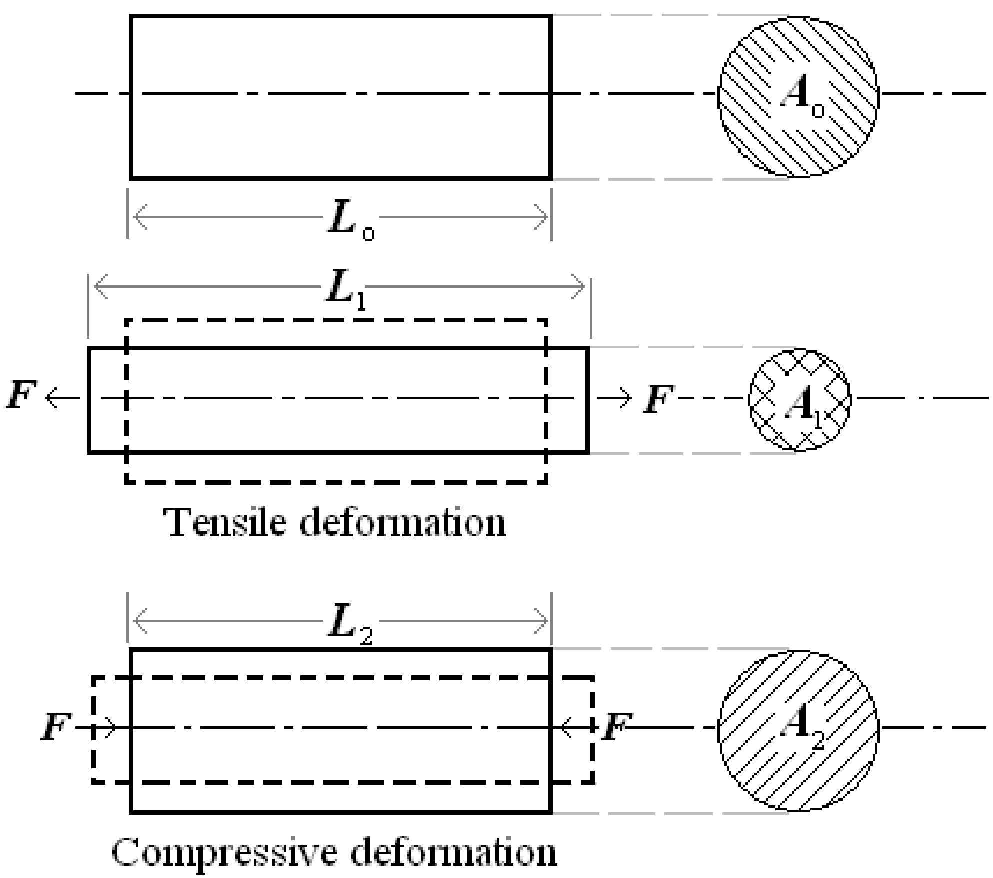
- True strain:

$$\varepsilon = \sum \frac{L_1 - L_0}{L_0} + \frac{L_2 - L_1}{L_1} + \frac{L_3 - L_2}{L_2} + \ldots \qquad \varepsilon = \int_{L_0}^{L} \frac{dL}{L} = \ln \frac{L}{L_0}$$

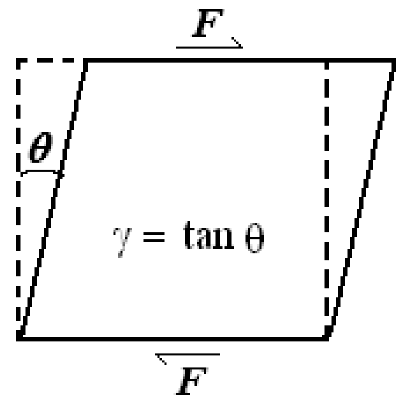
- True stress:

$$\sigma = \frac{P}{A} = \frac{P}{A_0}\frac{A_0}{A} = s(e+1)$$

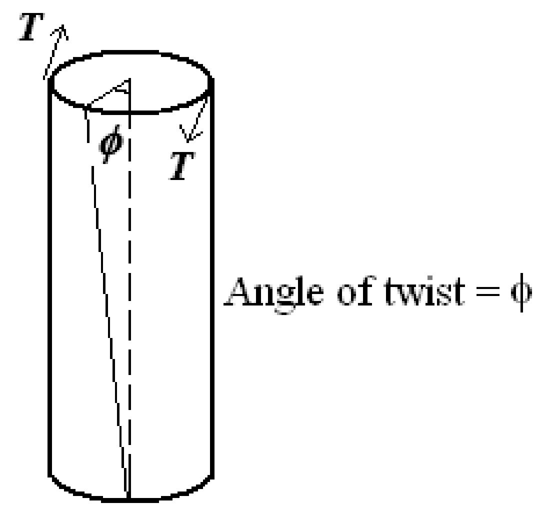
Different loads – Strains



Linear strains



Shear strain



Torsional strain

Elastic deformation

- A material under goes elastic deformation first followed by plastic deformation. The transition is not sharp in many instances.

- For most of the engineering materials, complete elastic deformation is characterized by strain proportional to stress. Proportionality constant is called *elastic modulus* or *Young's modulus*, *E*.

$$\sigma \propto \varepsilon \qquad\qquad \sigma = E\varepsilon$$

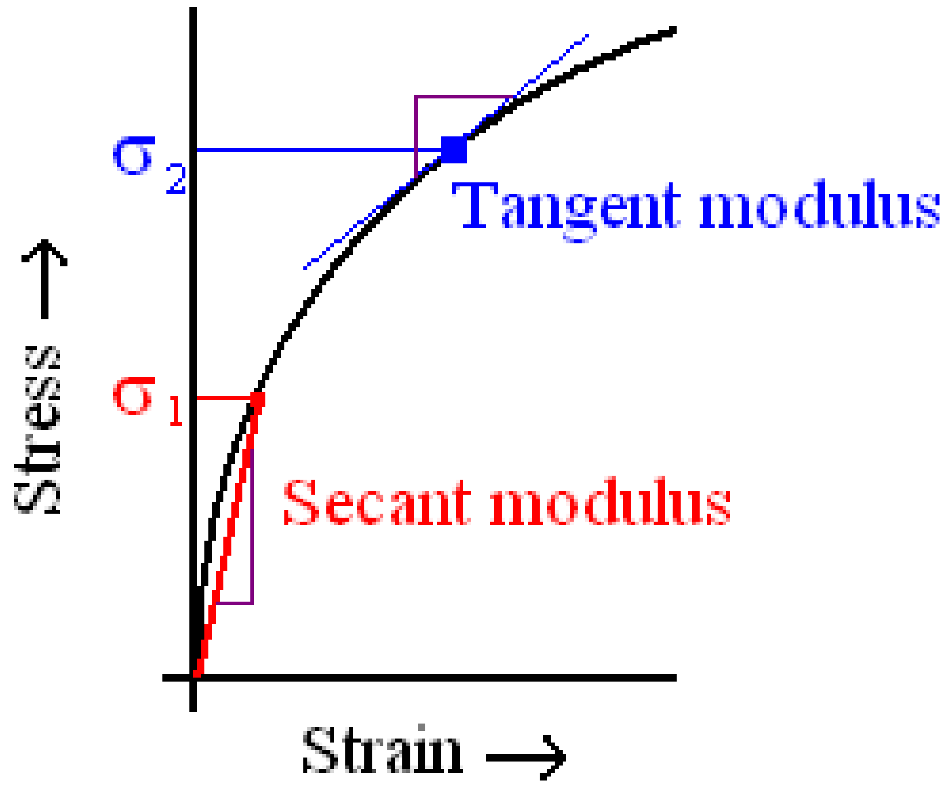
- Non-linear stress-strain relation is applicable for materials. E.g.: rubber.

Elastic deformation (contd…)

- For materials without linear stress-strain portion, either tangent or secant modulus is used in design calculations.

The tangent modulus is taken as the slope of stress-strain curve at some specified level.

Secant module represents the slope of secant drawn from the origin to some given point of the σ-ε curve.



Elastic deformation (contd…)

- Theoretical basis for elastic deformation – reversible displacements of atoms from their equilibrium positions – stretching of atomic bonds.
- Elastic moduli measures *stiffness* of material. It can also be a measure of resistance to separation of adjacent atoms.
- Elastic modulus = *fn* (inter-atomic forces)

  = *fn* (inter-atomic distance)

  = *fn* (crystal structure, orientation)

  => For single crystal elastic moduli are not isotropic.
- For a polycrystalline material, it is considered as isotropic.
- Elastic moduli slightly changes with temperature (decreases with increase in temperature).

Elastic deformation (contd…)

- Linear strain is always accompanied by lateral strain, to maintain volume constant.
- The ratio of lateral to linear strain is called Poisson's ratio (ν).
- Shear stresses and strains are related as $\tau = G\gamma$, where G is shear modulus or elastic modulus in shear.
- Bulk modulus or volumetric modulus of elasticity is defined as ratio between mean stress to volumetric strain. $K = \sigma_m/\Delta$
- All moduli are related through Poisson's ratio.

$$G = \frac{E}{2(1+\nu)} \qquad K = \frac{\sigma_m}{\Delta} = \frac{E}{3(1-2\nu)}$$

Plastic deformation

- Following the elastic deformation, material undergoes plastic deformation.
- Also characterized by relation between stress and strain at constant strain rate and temperature.
- Microscopically…it involves breaking atomic bonds, moving atoms, then restoration of bonds.
- Stress-Strain relation here is complex because of atomic plane movement, dislocation movement, and the obstacles they encounter.
- Crystalline solids deform by processes – slip and twinning in particular directions.
- Amorphous solids deform by viscous flow mechanism without any directionality.

Plastic deformation (contd…)

- Because of the complexity involved, theory of plasticity neglects the following effects:

  - Anelastic strain, which is time dependent recoverable strain.

  - Hysteresis behavior resulting from loading and un-loading of material.

  - Bauschinger effect – dependence of yield stress on loading path and direction.
- Equations relating stress and strain are called *constitutive equations*.
- A true stress-strain curve is called *flow curve* as it gives the stress required to cause the material to flow plastically to certain strain.

Plastic deformation (contd…)

- Because of the complexity involved, there have been many stress-strain relations proposed.

$$\sigma = fn(\varepsilon, \dot{\varepsilon}, T, microstructure)$$

$$\sigma = K\varepsilon^{n}$$

Strain hardening exponent, n = 0.1-0.5

$$\sigma = K\dot{\varepsilon}^{m}$$

Strain-rate sensitivity, m = 0.4-0.9

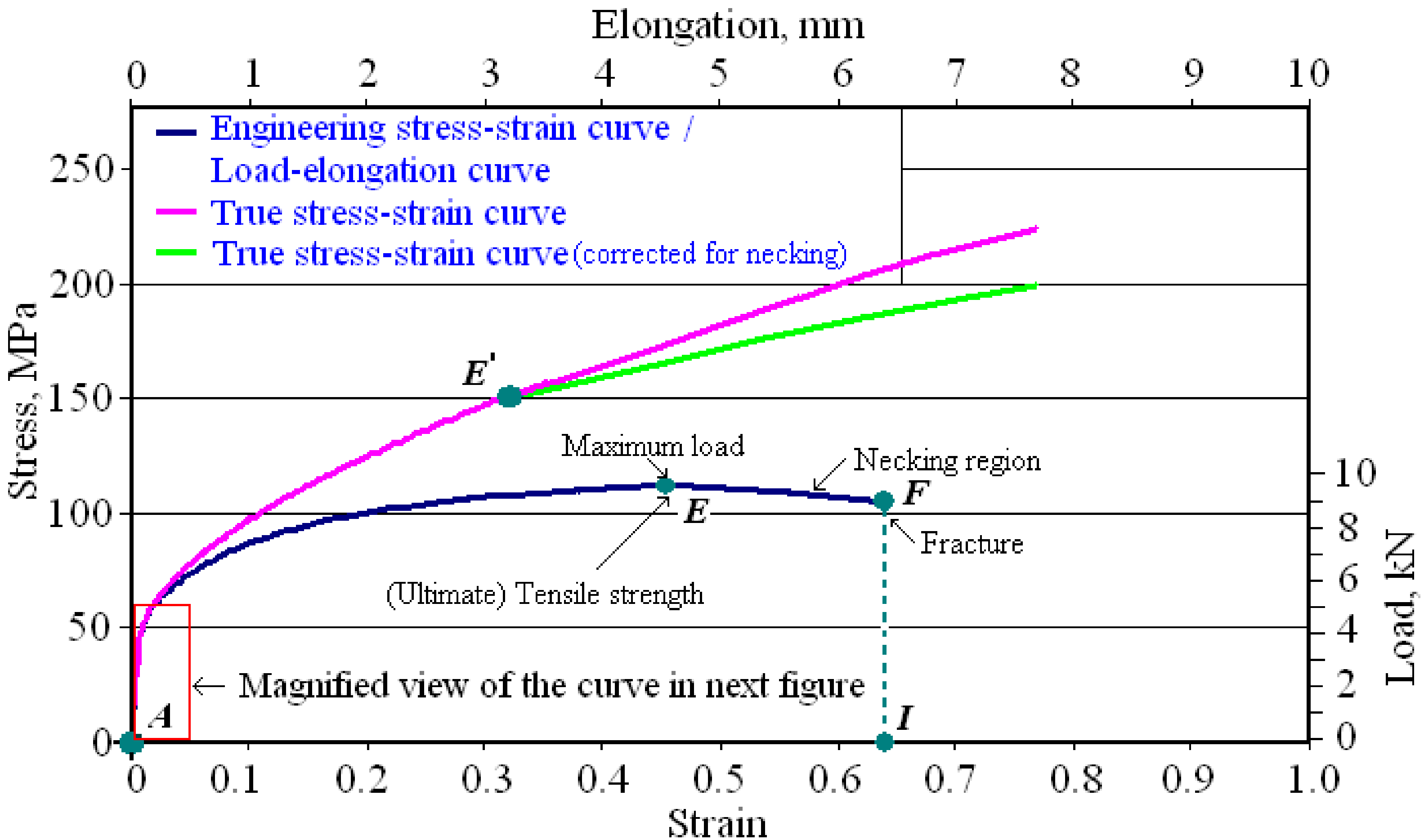
$$\sigma = K(\varepsilon_0 + \varepsilon)^{n}$$

Strain from previous work – ε_0

$$\sigma = \sigma_o + K\varepsilon^{n}$$

Yield strength – σ_0

Tensile stress-strain curve



A – Starting point

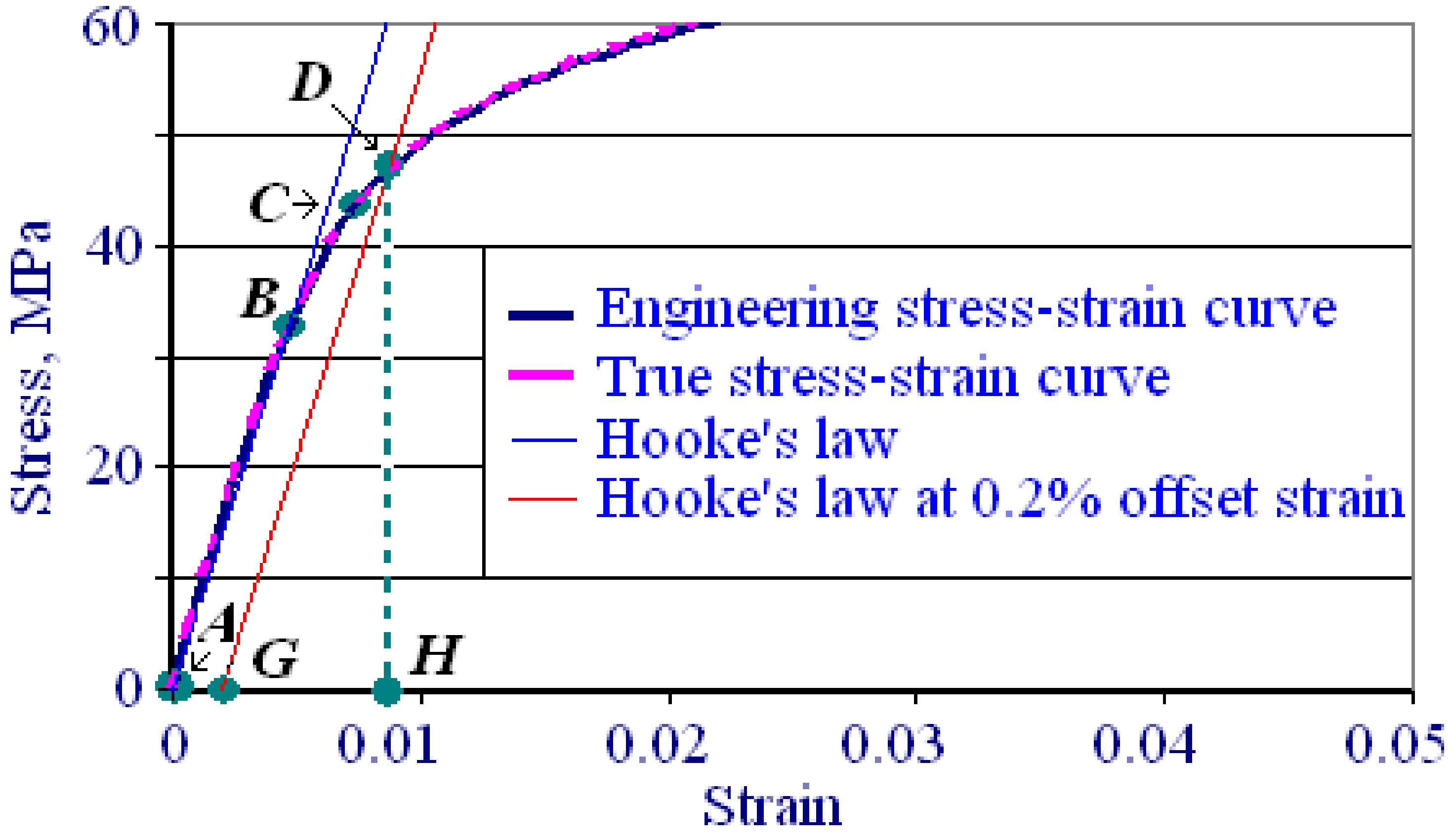
E– Tensile strength

E' – Corresponding to E on flow curve

F – Fracture point

I – Fracture strain

Tensile stress-strain curve (contd…)



A – Starting point

B – Proportional limit

C – Elastic limit

D – Yield limit

G – 0.2% offset strain

H – Yield strain

Tensile stress-strain curve (contd…)

- Apart from different strains and strength points, two other important parameters can be deduced from the curve are – resilience and toughness.
- Resilience (U_r) – ability to absorb energy under elastic deformation
- Toughness (U_t) – ability to absorb energy under loading involving plastic deformation. Represents combination of both strength and ductility.

$$U_r = \frac{1}{2} s_0 e_0 = \frac{1}{2} s_0 \frac{s_0}{E} = \frac{s_0^2}{2E}$$ *area* ADH

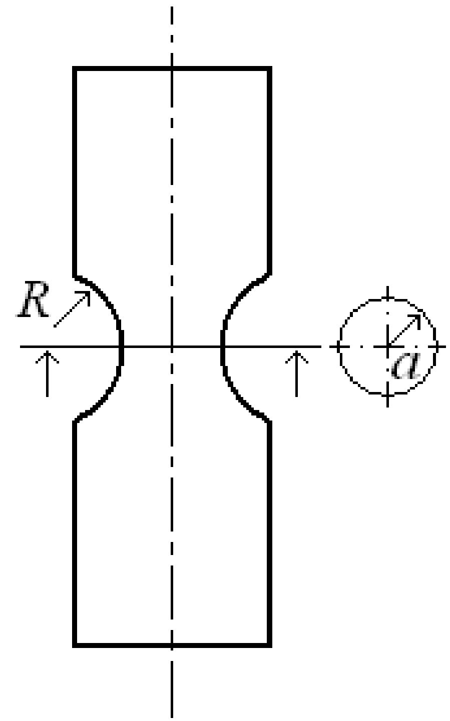
$$U_t \approx s_u e_f \approx \frac{s_0 + s_u}{2} e_f$$ *area* AEFI

$$U_t \approx \frac{2}{3} s_u e_f$$ *(for brittle materials)*

Yielding under multi-axial stress

- With on-set of necking, uni-axial stress condition turns into tri-axial stress as geometry changes tales place. Thus flow curve need to be corrected from a point corresponding to tensile strength. Correction has been proposed by Bridgman.

$$\sigma = \frac{(\sigma_x)_{avg}}{(1+2R/a)\ln(1+a/2R)}$$



where

$(\sigma_x)_{avg}$ measured stress in the axial direction,

a – smallest radius in the neck region,

R – radius of the curvature of neck

Yield criteria

- von Mises or Distortion energy criterion:

  yielding occurs once second invariant of stress deviator (J_2) reaches a critical value. In other terms, yield starts once the distortion energy reaches a critical value.

$$J_2 = k^2 \qquad J_2 = \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

Under uni-axial tension, $\sigma_1 = \sigma_0$, *and* $\sigma_2 = \sigma_3 = 0$

$$\frac{1}{6}(\sigma_0^2 + \sigma_0^2) = k^2 \Rightarrow \sigma_0 = \sqrt{3}k$$

$$\Rightarrow \sigma_0 = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

$$k = \frac{1}{\sqrt{3}}\sigma_0 = 0.577\sigma_0$$

where k – yield stress under shear

Yield criteria (contd…)

- Tresca or Maximum shear stress criterion

  yielding occurs once the maximum shear stress of the stress system equals shear stress under uni-axial stress.

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2}$$

Under uni-axial tension, $\sigma_1 = \sigma_0$, *and* $\sigma_2 = \sigma_3 = 0$

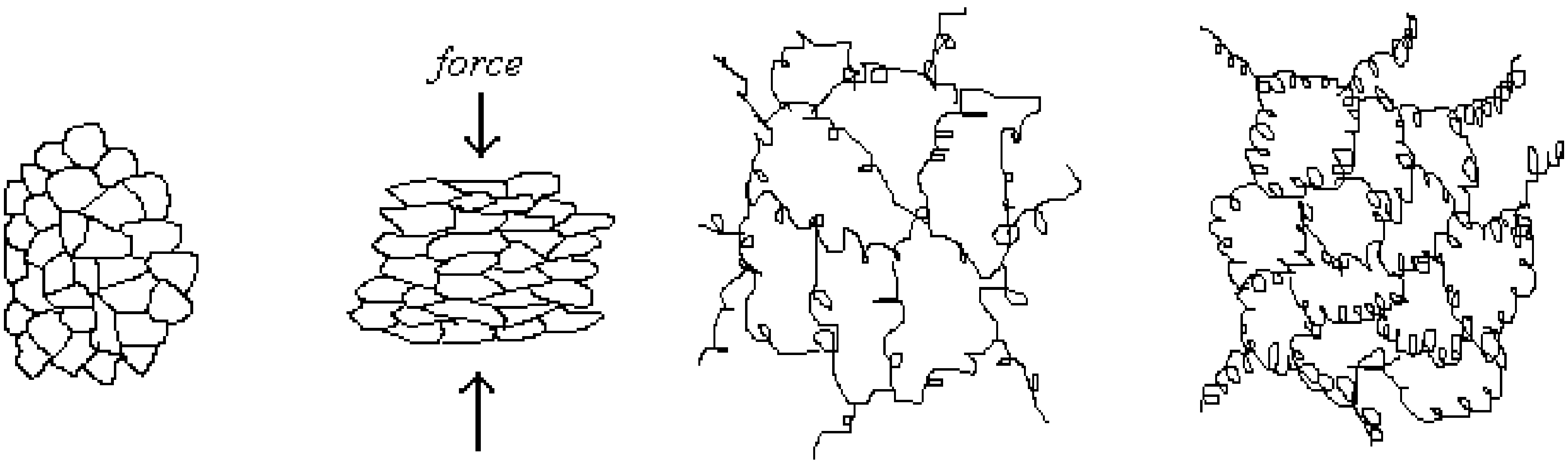
$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \tau_0 = \frac{\sigma_0}{2} \Rightarrow \sigma_1 - \sigma_3 = \sigma_0$$

Under pure shear stress conditions ($\sigma_1 = - \sigma_3 = k$, $\sigma_2 = 0$)

$$k = \frac{\sigma_1 - \sigma_3}{2} = \frac{1}{2}\sigma_0$$

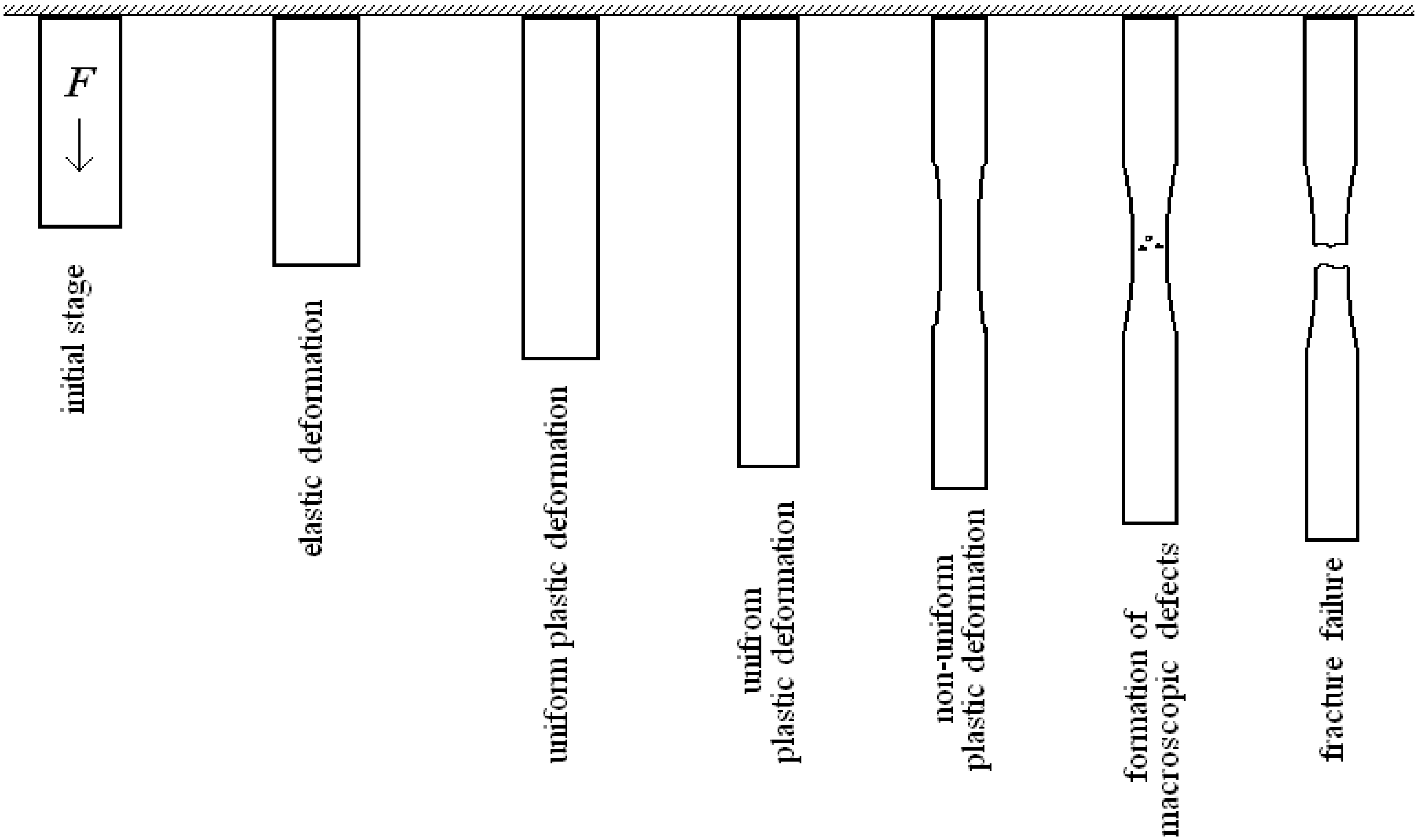
Macroscopic aspects – Plastic deformation

- As a result of plastic deformation (Dislocation generation, movement and (re-)arrangement), following observations can be made at macroscopic level:
  dimensional changes
  change in grain shape
  formation of cell structure in a grain



equi-axed grains *deformed grains* *initial cell structure* *denser cell structure*

Macroscopic aspects – Plastic deformation (contd…)



Property variability

- Scatter in measured properties of engineering materials is inevitable because of number of factors such as:

  test method

  specimen fabrication procedure

  operator bias

  apparatus calibration, etc.

Average value of x over n samples.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Property variability measure – Standard deviation

$$s = \left[\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} \right]^{1/2}$$

Scatter limits:

$$\bar{x} - s, \quad \bar{x} + s$$

Design consideration

- To account for property variability and unexpected failure, designers need to consider tailored property values. Parameters for tailoring: safety factor (N) *and* design factor (N'). Both parameters take values greater than unity only.

E.g.: Yield strength

$$\sigma_w = \sigma_y / N \qquad \sigma_d = N'\sigma_c$$

where σ_w – working stress

σ_y – yield strength

σ_d – design stress

σ_c – calculated stress

Design consideration (contd…)

- Values for *N* ranges around: 1.2 *to* 4.0.

- Higher the value of N, lesser will the design efficiency i.e. either too much material *or* a material having a higher than necessary strength will be used.

- Selection of *N* will depend on a number of factors:
  economics
  previous experience
  the accuracy with which mechanical forces
  material properties
  the consequences of failure in terms of loss of life or property damage.